

7.2 notes

PROBABILITY RULES!

7.2: Probability

Probability Models- 2 parts:

- * A list of all possible outcomes.
- * The probability of each of those outcomes.

EXAMPLE: M&Ms

Color	Prob.
Brown	30%
Red	20%
Yellow	20%
Green	10%
Orange	10%
Blue	?

10%.

Sample Space- All possible outcomes

Examples:

What is the sample space when I roll 2 dice and look at the sum?

2, 3, 4, 5, 6, ..., 12

What is the sample space for the Plinko expt?

0, 100, 300, 400, 500, 200

What about the "Let's make a Deal" experiment?

Win, Lose

Event- An outcome, or set of outcomes.

Ex: When rolling one die, events could be:

Evens, a 5, less than 4, etc.

Probability Notation:

- A, B, C, etc. = **events** also use words H Heart

- $P(A)$ = ^{event} The probability of event A occurring $P(H) = \frac{1}{4}$
 $P(\text{Heart})$

$$\frac{\text{\# outcomes in A}}{\text{total possible outcomes}} = \text{decimal percent}$$

- S = the Sample Space (all possible outcomes)

$$P(S_p) = P(\text{Spade})$$

- A^c = The complement of A

Event A **NOT** happening

$$= 1 - P(A)$$

$$P(H^c) = \frac{3}{4}$$

Basic Probability Rules

$$1) \frac{0}{0} \leq P(A) \leq \frac{1}{100\%}$$

$$2) P(S) = 1 \text{ (the sum of the probabilities of all outcomes = 1)}$$

$$3) P(A^c) = 1 - P(A)$$

$$P(\text{Red or Spade}) \\ \frac{1}{2} + \frac{1}{4} = \left(\frac{3}{4}\right)$$

4) Basic addition rule:

* When 2 events have no outcomes in common =

MUTUALLY EXCLUSIVE or DISJOINT

* The probability of one event or the other event occurring =

The sum of their individual probabilities

$$* P(A \text{ or } B) = P(A) + P(B)$$

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun?

$$P(H) = 30\% = 0.30 \quad P(H^c) = 0.70 = 70\%$$

Example 2: If there are only 8 different blood types, fill in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	0.06	0.07	0.1	0.11

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, “UNC’s probability of winning is twice Duke’s. NC State and UVA each have probability 0.1 of winning, but Duke’s probability is three times that. Nobody else has a chance.” Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

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$$P(\text{UNC}) = 0.60$$

$$P(\text{Duke}) = 0.30$$

$$P(\text{NC St.}) = 0.10$$

$$P(\text{UVA}) = 0.10$$

$$1.1$$

Example 4: Suppose 40% of cars in your area are manufactured in the United States, 30% in Japan, and 10% in Germany.

Probability Model:

Car Origin	US	Japan	Germany	Other
Probability	40%	30%	10%	20%

If a car is selected at random find the probability that:

a) The car is not US made.

$$P(US^c) = P(\text{not US}) = 60\%$$

b) The car is made in Japan or Germany.

$$P(\text{Japan or Germany}) = P(J \text{ or } G) = 40\%$$

c) The car is made in the US or Japan.

$$P(US \text{ or } J) = 70\%$$

d) The car is neither German nor US.

$$P(G^c \text{ and } US^c) = 50\%$$

e) The car is not made in US nor Japan nor Germany.

$$P(US^c \text{ and } J^c \text{ and } G^c) = 20\% = P(\text{Other})$$

EXAMPLE: There are only 8 different blood types, given below

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	0.06	0.07	0.1	0.11

What is the probability of being either Type A+ or B-?

$$P(A^+ \text{ or } B^-) = 0.33$$

What is the probability of being either Type O- or O+?

$$P(O) = 0.21$$

What is the probability of being either Type AB+ or A+?

$$P(AB^+ \text{ or } A^+) = 0.22$$

What is the probability of not having Type A blood?

$$P(A^c) = 0.70 = 70\%$$

What is the probability of having some B blood?

$$P(AB \text{ or } B) = P(B^+ \text{ or } B^- \text{ or } AB^+ \text{ or } AB^-) = 0.49$$

What is the probability of having type O and type A blood?

$$P(O \text{ and } A) = 0$$

ADDITON RULE:

Example: We are picking one card out of a standard 52-card deck (no jokers).

- a) What is the probability of picking a diamond?

$$P(D) = 13/52$$

- b) What is the probability of picking a 3?

$$P(3) = 4/52$$

- c) What is the probability of picking a diamond or a 3?

$$\begin{aligned} P(D \text{ or } 3) &= 13/52 + 4/52 - 1/52 \\ &= 16/52 \end{aligned}$$

ADDITION RULE:

To find the probability of event A OR event B happening....

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Examples: Picking cards, etc.

SPECIAL CASE:

Mutually exclusive = Disjoint = When 2 events (A, B)
have no outcomes in common

$$P(A \text{ and } B) = 0$$

When two events are disjoint...

$$P(A \text{ or } B) = P(A) + P(B)$$

Examples: car types, blood types, etc.

Example: We are picking one card out of a standard 52-card deck

- a) What is the probability of picking a 4 or a Club?

$$P(4 \text{ or Club}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

- b) What is the probability of picking a 4 or a Jack?

$$P(4 \text{ or Jack}) = \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52}$$

- c) What is the probability of picking a red card or a Queen?

$$P(\text{Red or Q}) = \frac{28}{52}$$

- d) What is the probability of picking a Spade or a red card?

$$P(\text{Sp or R}) = \frac{39}{52}$$

- e) What is the probability of picking a black card or a face card?

$$P(\text{Bl or face}) = \frac{32}{52}$$

Return of the two-way tables:

Example 1: A random sample of students a local high school had the following results.
Find the probability that a randomly selected student is:

a) A male.

$$P(M) = \frac{41}{100} = 0.41$$

b) Plays an instrument.

$$P(Y) = P(I) = \frac{47}{100} = 0.47$$

c) A male and plays an instrument

$$P(M \text{ and } I) = \frac{15}{100} = 0.15$$

d) A male or plays an instrument.

$$P(M \text{ or } I) = \frac{41}{100} + \frac{47}{100} - \frac{15}{100} = \frac{73}{100}$$

e) A female or plays an instrument.

$$P(F \text{ or } I) = 59 + 47 - 32 = \frac{74}{100}$$

f) A female and doesn't play an instrument.

$$P(F \text{ and } I^c) = \frac{27}{100}$$

Sex	Do you play an instrument?		Total
	Yes	No	
Male	15	26	41
Female	32	27	59
Total	47	53	100

Try Example 2: We take a random sample of adults and record their gender and political preference. Find the probability that randomly selected adult is:

- 1) Male?

$$P(M) = 115/192$$

- 2) Female and liberal?

$$P(F \text{ and } L) = 35/192$$

- 3) Moderate or male?

$$P(M \text{ or } M) = 151/192$$

- 4) Male and conservative?

$$P(M \text{ and } C) = 21/192$$

- 5) Liberal?

$$P(L) = 85/192$$

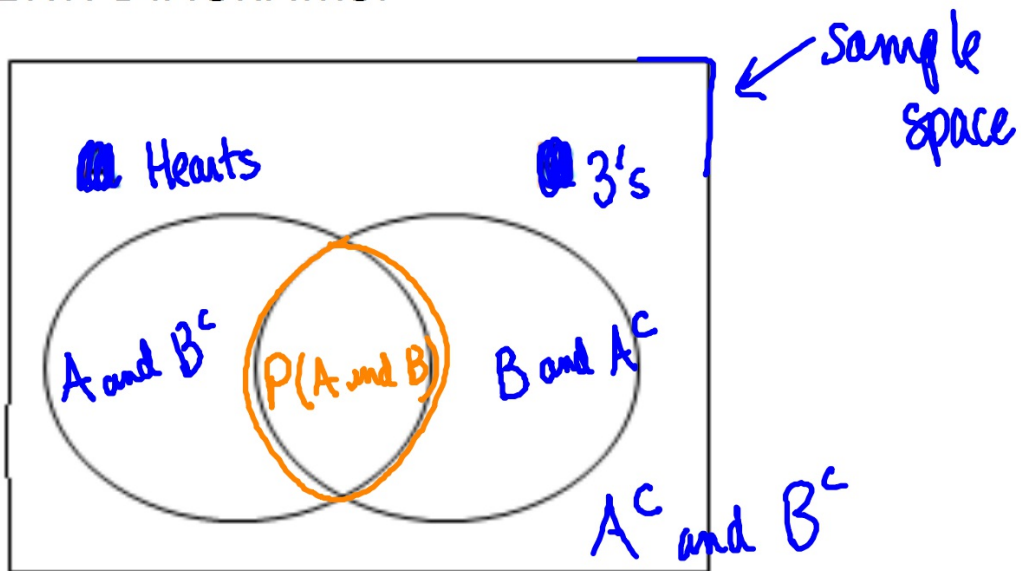
- 6) Female and not conservative?

$$P(F \text{ and } C^c) = 71/192$$

- 7) ~~Male or moderate?~~

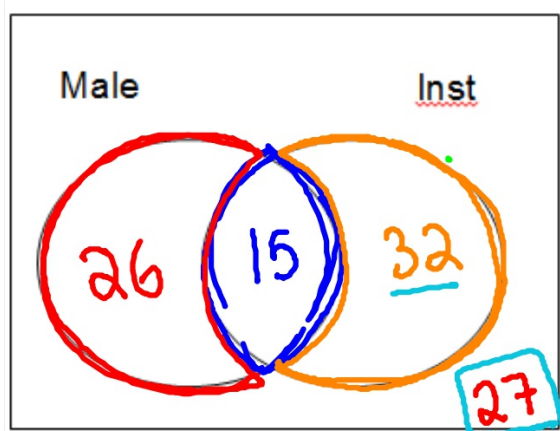
	Political Preference			total
	Liberal	Moderate	Conservative	
Gender Female	35	36	6	77
Male	50	44	21	115
total	85	80	27	192

VENN DIAGRAMS:



Venn Diagrams and Two-Way Tables:

Example 1: Given the table from before, create the matching Venn Diagram



Sex	Do you play an instrument?		Total
	Yes	No	
Male	15	26	41
Female	32	27	59
Total	47	53	100

What is the probability that a randomly selected person is:

a) A male and plays an instrument

$$P(M \text{ and } I) = \frac{15}{100}$$

c) A female or plays an instrument.

$$P(F \text{ or } I) = \frac{74}{100}$$

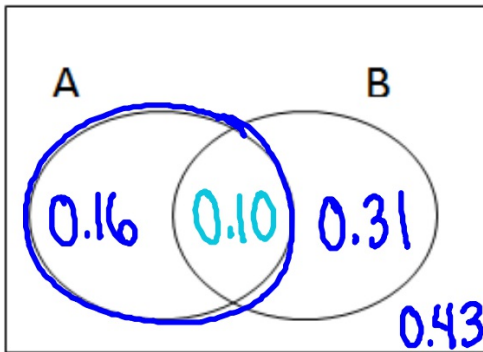
b) A male or plays an instrument.

$$P(M \text{ or } I) = \frac{73}{100}$$

d) A female and doesn't play an instrument.

$$P(F \text{ and } I^c) = \frac{27}{100}$$

Example 2: If $P(A) = 0.26$, $P(B) = 0.41$ and $P(A \text{ and } B) = 0.10$



	B	Not B	Total
A	0.10	0.16	0.26
Not A	0.31	0.43	0.74
Total	0.41	0.59	1.00

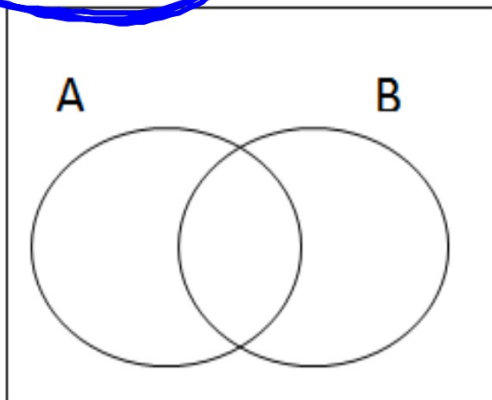
a) $P(A \text{ or } B) = 0.57$

b) $P(\text{not } A \text{ and } B) = 0.31$

c) $P(\text{not } A \text{ or } B) = 0.84$

d) $P(A \text{ or not } B) = 0.69$

Example 3: 1) $P(A) = 0.56$, $P(B) = 0.68$ and $P(A \text{ and } B) = 0.32$



	B	Not B	Total
A			
Not A			
Total			

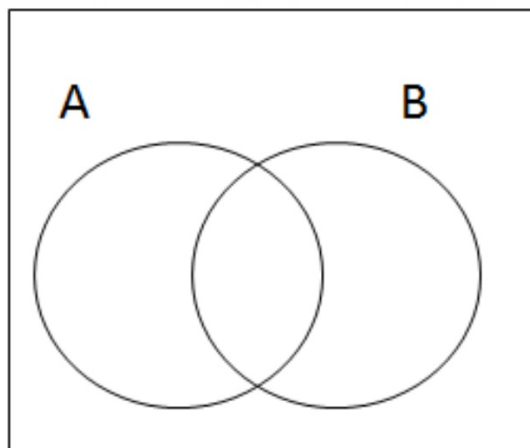
a) $P(A \text{ or } B) =$

b) $P(\text{not } A \text{ and not } B) =$

c) $P(A \text{ and not } B) =$

d) $P(\text{not } A \text{ or } B) =$

Example 4: If $P(A) = 0.47$; $P(B) = 0.19$ and $P(A \text{ and } B) = 0.09$



	B	Not B	Total
A			
Not A			
Total			

a) $P(A \text{ or } B) =$

b) $P(\text{not } A \text{ and not } B) =$

c) $P(A \text{ and not } B) =$

d) $P(\text{not } A \text{ or } B) =$

Example 5: At a local university a sample is taken of the students. It is found that out of 200 students 140 were undergraduates, 90 were males, and 70 were male undergraduates. Create either a Venn Diagram or a Table to help you answer the questions below.

What is the probability that a randomly selected student was:

- a. Graduate student?
- b. Female?
- c. Male and Graduate student?
- d. Male or Undergraduate student?
- e. Female and a Graduate student?
- f. Female or a Graduate student?

Example 6: At a local track meet 62% of the participants were girls, 29% of the participants were from private schools, and 18% of the participants were girls from private schools. Create either a Venn Diagram or a Table to help you answer the questions below.

- a. What is the probability that a randomly selected student was a girl or from a private school?
- b. What is the probability that randomly selected student was a girl and not from a private school?
- c. What is the probability that a randomly selected student was not a girl and from a private school?
- d. What is the probability that a randomly selected student was not a girl or not from a private school?

Practice:

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p. 337 #33, 35, 36

p. 343 #39, 43